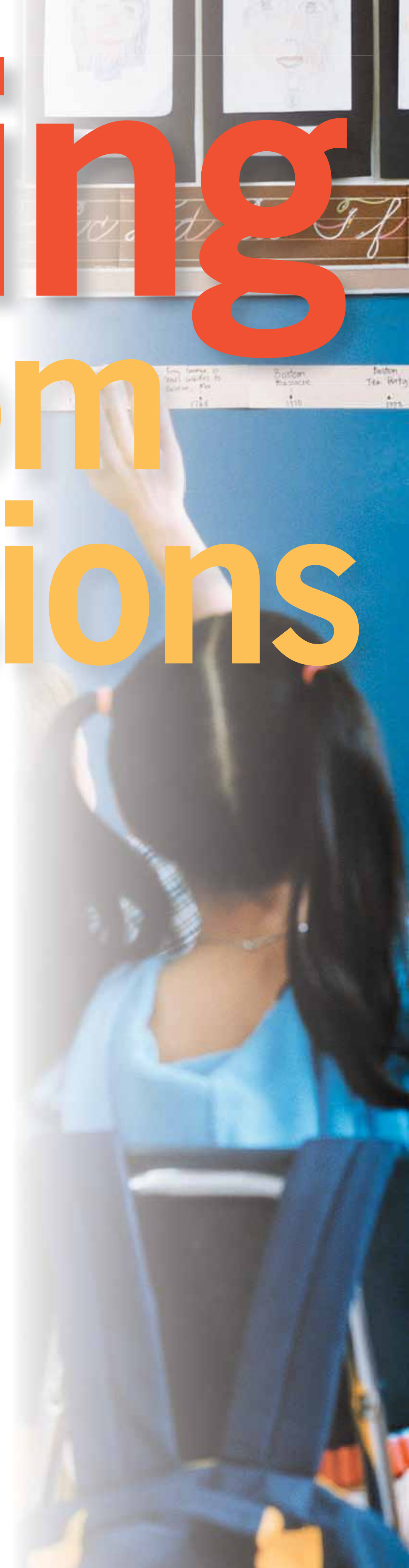
Leading Classroom Discussions



Students develop ownership and increase their understanding of mathematics when they are allowed to discuss alternative perspectives.

Gloriana González and Anna F. DeJarnette

Classroom discourse is a valuable teaching and learning tool. Discussions allow students to improve their communication and reasoning skills in mathematics and help teachers assess students' understanding of mathematical ideas (NCTM 2009). To get the greatest benefit from discussion, teachers must elicit student thinking, listen carefully to their ideas, and ask them to clarify and justify their thoughts.

This article describes examples of discursive moves that a teacher, whom we call Mr. Jenkins, performed in several seventh-grade prealgebra classes. These examples illustrate the opportunities for learning that the teacher provided when leading a class discussion. In all the episodes, Jenkins was leading a discussion of a homework assignment about functions and linear equations, which occurred before the topics were formally taught.

We focus on two discursive moves that Jenkins performed when leading the discussion: *request an explanation* and *orient* (Herbst 2011). The names of the different types of discursive moves convey the types of actions that Jenkins performed to promote students' reasoning and sense making. We also propose alternative statements or questions that a teacher may pose in these situations.

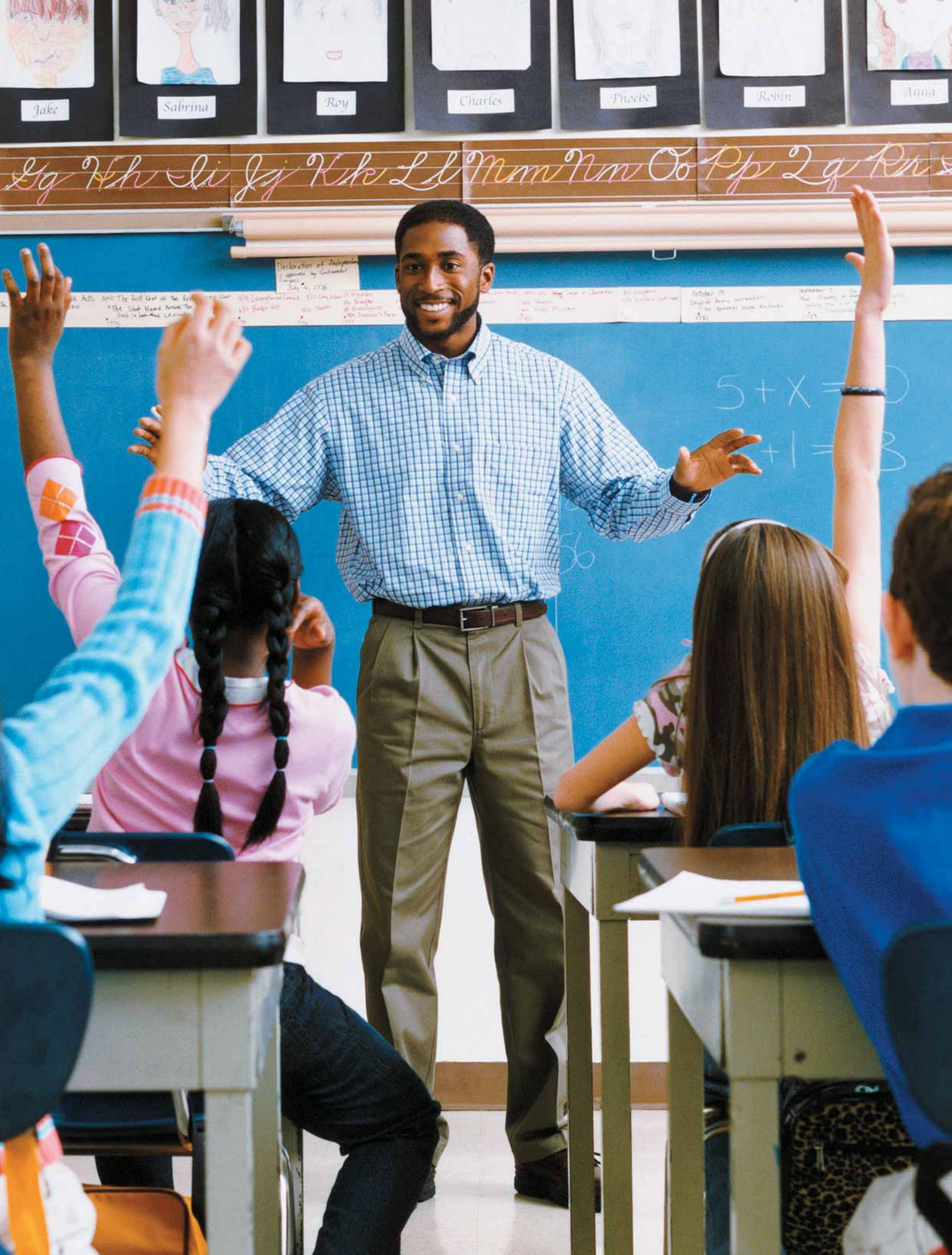

Jake
Sabrina
Roy
Charles
Phoebe
Robin
Anna

Fig. 1 This problem explored growth rates and the intersection points of two lines.

Consider the equations
$y_1 = 2x - 4$ and $y_2 = 3x + 1$.

a. Make a table of values for each equation for values of x ranging from $x = 1$ to $x = 10$.

b. Which equation grows faster? How do you know?

c. Do the two lines ever intersect? How do you know?

Fig. 2 The linear equations $y = 2x - 4$ and $y = 3x + 1$ were produced in table form.

x	$2x - 4$	y
1	2(1) – 4	–2
2	2(2) – 4	0
3	2(3) – 4	2
4	2(4) – 4	4
5	2(5) – 4	6
6	2(6) – 4	8
7	2(7) – 4	10
8	2(8) – 4	12
9	2(9) – 4	14
10	2(10) – 4	16

x	$3x + 1$	y
1	3(1) + 1	4
2	3(2) + 1	7
3	3(3) + 1	10
4	3(4) + 1	13
5	3(5) + 1	16
6	3(6) + 1	19
7	3(7) + 1	22
8	3(8) + 1	25
9	3(9) + 1	28
10	3(10) + 1	31

Fig. 3 This problem asked students to compare the graphs of two linear equations.

For each of the following two equations, make a table of values and then graph each equation. How are the graphs similar? Different?

a. $y = 2x + 7$
b. $y = 2(x + 3)$

Fig. 4 Nancy produced these tables of values (see episode 2).

x	$2x + 7$	y
–1	2(–1) + 7	5
1	2(1) + 7	9
0	2(0) + 7	7
2	2(2) + 7	11

x	$2(x + 3)$	y
–1	2(–1 + 3)	4
1	2(1 + 3)	8
0	2(0 + 3)	6
2	2(2 + 3)	10

REQUEST AN EXPLANATION

Teachers can request an explanation from students to elicit their reasoning about the claims they make (see, e.g., Chazan and Sandow 2011; Forman, McCormick, and Donato 1998; González and DeJarnette 2012). Rather than accepting or rejecting a student's answer, a teacher may request that the student provide more insight into his or her reasoning.

Episode 1

Jenkins and a student were discussing a problem in which students had to determine which of two linear functions would grow faster based on a table of values they had created (see **fig. 1** and **fig. 2**). Jenkins asked a student to provide an explanation.

Jenkins: Now, one of the questions was, which equation actually increases quicker? What do you think, Jason?
Jason: Um, the one on the right.
Jenkins: Now why does it increase quicker? Why did you say that?
Jason: Because for every one the x goes up, the y goes up by 3. So, like, if you look at 10 on both of 'em, the one on the right's obviously bigger.

In this dialogue, Jenkins asked Jason to explain why the second equation grew faster. By asking, "Why did you say that?" Jenkins created an opportunity to understand the reasoning behind Jason's answer. This reasoning was especially significant in this case because although Jason's answer was correct, his reasoning was only partially correct. His explanation revealed a misconception about the growth of the two equations. Jason's strategy of looking at the x-coordinate of 10 on each equation displayed a misconception about the difference between analyzing the values of x and y and the growth rate of the equation.

This episode from Jenkins's class highlights how important "request an explanation" is to foster students' reasoning skills. Affirming or disregarding an answer will not reveal the student's thinking, but requesting an explanation will give the teacher an opportunity to understand the student's reasoning process. Moreover, Jenkins used Jason's explanation as an opportunity to help Jason and the entire class improve their mathematical understanding of growth rates.

Episode 2

Students had to construct a table of values for two linear functions with the same slope but with different y-intercepts. The students also had to graph the functions and compare the graphs (see **fig. 3**). This episode illustrates how students can also "request an explanation" from their peers. Nancy, a

student who went to the board to show her solution to the problem, had made a table of values. She chose –1, 1, 0, and 2 (in that order) as *x*-values for the two equations (see **fig. 4**).

After Nancy's presentation, Jenkins prompted the class to ask questions about her explanation. A student asked her why she had chosen those values for *x*. Nancy said, "Well, that's what we learned about last year, 'cuz it's like the standard easy numbers to put for *x*." Jenkins asked, "So, I can just choose any numbers I want for *x*?" Nancy said, "No, well, you can, but these are the easiest ones to do." Jenkins repeated Nancy's answer and asked the class if they agreed or disagreed. Students voiced that they agreed with Nancy's statement. At this point, Jenkins asked, "Why do you agree?" and engaged students in a discussion about the reasons for choosing specific values for *x*. He then summarized the discussion by stating that when a specific set of values for *x* has not been provided, then one can choose values for *x*.

In this episode, Jenkins gave an example of an alternative way to elicit an explanation. Rather than pressing Nancy to explain her reasoning herself, Jenkins gave the rest of the class the opportunity to decide whether they wanted further explanation. In suspending his own judgment of Nancy's answer and allowing his students to request more explanation, Jenkins enabled the class to lead the classroom discussion and to facilitate their own communication and reasoning with one another. Jenkins's summary occurred when he had enabled students to work by themselves on the solution to a question. Students were discussing the domain of a linear function before being formally introduced to the concept of domain.

We find that episode 2 shows how a teacher can provide opportunities for students to further develop their agency. In other words, it is important for students to have the capacity to inquire into one another's mathematical justifications and develop the habit of engaging in mathematical discussions. The teacher demonstrated his expectation that students were to listen to one another and be critical of others' comments. At the same time, the teacher held students accountable for responding to others' comments and questions. A teacher enabling students to request explanations from one another is crucial for building a mathematical community centered on students' active engagement with mathematics.

It is important for students to inquire into one another's mathematical justifications and engage in mathematical discussions.

ORIENT DIFFERENT APPROACHES TO A PROBLEM

In a classroom discussion, students can voice different ideas. However, it may be difficult for them to keep track of what other students have said. The move *orient* involves grouping many comments into a limited number of perspectives and allowing students to compare those perspectives (Ghousseini 2009; Chapin, O'Connor, and Anderson 2003). For example, ten different students may be participating in a discussion in which the underlying mathematical ideas can be summarized and grouped into two or three perspectives. By orienting, a teacher clusters the ten different comments into a manageable number of ideas for the class to consider. Two functions of orienting are to (1) help students make sense of different approaches to solving a problem and (2) discuss different solutions to a problem.

Episode 3

We saw an example of orienting in another discussion about comparing the two linear equations in **figure 1.** Students had to compare the growth rate of two linear functions and anticipate whether the graphs of those functions would intersect from studying the table of values. The following episode shows how Jenkins requested explanations and also oriented students' discussion. This combination of moves enabled Jenkins to ask students to assume different positions regarding the answer to the problem as well as their justification for their answers. This discussion occurred after they had talked about the table of values representing each equation (see **fig. 2**).

Jenkins: Do the two lines, if you were to make a graph, ever actually intersect? Tim, what do you think?
Tim: No. [Because] if you make a graph. I didn't make a graph but. . . .
Jenkins: OK. I didn't say you had to make a graph.
Tim: Oh. I don't know. OK.
Jenkins: Jason, what do you think?
Jason: Um, I think no.
Jenkins: Anybody else?
Renee: I said yes because on both sides you have some of the same numbers.
Jenkins: Ahh, OK, let's look at this. What you are saying is, saying there is a 10 right here, right? As a *y*-value, and there's a 10 right here as a *y*-value. Since they have the same value of *y*, they must at some point meet.

In the episode, Jenkins combined two moves: requesting an explanation and orienting. He asked students to justify their answers *and* elicited different answers from students to establish that they had different positions. Some students (Tim and Jason) stated

that the lines would not intersect. In contrast, another student (Renee) stated that the lines would intersect. The different opinions as to whether the lines would intersect or not gave an opportunity for Renee to state the reasons behind her answer.

We find that this discussion could have provided opportunities for students to develop their reasoning and sense making in several ways. First, the teacher's question pushed students from using a table of values to visualizing the graph of the functions that the table of values represented. Therefore, even though the students had not constructed the graph, the teacher asked them to think about the graphs of the two linear functions as if these were a system of equations and to consider whether the lines intersected or not. Second, the teacher refrained from evaluating the students' answers right away.

A pattern of interaction called IRE (Cazden 2001) exists in many classrooms: The teacher initiates the pattern by asking a question (I), the student replies to that question (R), and the teacher evaluates the student's answer (E). It can be very difficult for teachers to depart from this pattern of interaction and avoid evaluating a student immediately after he or she has given a response. However, Jenkins did not evaluate Jason's answer when he said incorrectly that the lines would not intersect. Instead, he deferred to the rest of the class when he asked, "Anybody else?"

As a result, he was able to provide opportunities for other students to think about and to evaluate the original answer. Finally, Jenkins's decision to open up the discussion enabled him to elicit a misconception that students possessed about the solution to a system of linear equations: If the y-values are repeated, then the lines must intersect. This was a crucial moment in the discussion, because Jenkins was able to tap into the students' misconception and start a new discussion to address that misconception.

The discursive moves that Jenkins performed in this episode achieved the purpose of eliciting a misconception. Jenkins invited several students to participate during the discussion. As a result, there were different ideas for the class to consider. Then, Jenkins asked students to support their answers with an explanation. This was particularly important because Renee had stated the correct answer but had provided an incorrect justification.

Finally, Jenkins revoiced Renee's idea with the specific case of $y = 10$, which allowed the entire class to see with a concrete example what the student had presented as a generalized solution. He could have asked Renee to further explain what she meant when she said that both sides had some of the same numbers. Through further discussion, Renee might elaborate on her idea that because the two tables included some of the same y-values, the graphs would intersect at those y-values.

As a possible follow-up to this episode, the teacher could show a graph of the two functions to explore the possibility that the functions would intersect at the y-values that appeared in both graphs. This discussion could enable students to establish connections between multiple representations of a linear function, specifically, the formula, the table of values, and the graph. If students had graphing calculators, they could spend time discussing the graphs with their partners or in small groups.

Alternatively the teacher could project the graphs on a large screen and discuss growth rates using the graphs. The discussion of the graph could help students develop new arguments about what it means for two graphs to intersect, creating an opportunity to eradicate the misconception that Renee revealed. This discussion might provide more opportunities for reasoning and sense making before the formal introduction of linear functions.

Episode 4

As stated earlier, orienting could be performed when students voice ideas that appear to be different but actually are mathematically similar. We created a hypothetical scenario with the same problem as in episode 3, focusing on the question of whether two lines would intersect (see **fig. 3**). Various students provide different ideas when answering the question, and the teacher orients their ideas. The teacher asked the class if the two lines ever intersected and how they could be sure. Student responses included these:

- *Lilly:* I said that they do not intersect. . . . Because there are not the same coordinates in the two tables.
- *Akuti:* I said that they intersect. . . . Because if you continue the pattern in the table, you get to a point where the two *x*s and the two *y*s are the same.

Like Akuti, others believed the lines intersected and supported their conclusion:

- *Naomi:* Well, the first pattern in the table was adding 2. But it was making it bigger, and I needed to go backward. I started subtracting 2 until I got to –14. I did the same with the other pattern but subtracting 3, and I also got to –14.
- *Martin:* I made an equation: $2x - 4 = 3x + 1$. I got $x = -5$.

The teacher summarizes the ideas of the students in this way:

> We have several ideas here. Lilly said that the tables of values do not show the same coordinates for the two equations, and that's true. Most of you seem to agree that the lines will intersect. But I hear that to find the point of intersection, you had to expand the tables with more values. Naomi wanted to pay attention to the coordinates when x was negative. Akuti and Naomi used the pattern in the table of values to find a coordinate where the x- and y-values are the same. Martin did something different. He assumed that the y-values had to be the same and created an equation where y_1 equals y_2. Naomi had –14, and Martin had –5. Let's try to discuss the meaning of those numbers to answer the big question of whether the lines will intersect or not.

This example illustrates that a teacher can use orienting to position several different perspectives about the solutions to a problem. At one level, there were different opinions as to whether the lines intersected or not. Lilly asserted that the lines do not intersect. In contrast, Akuti, Naomi, and Martin said that the lines intersect. At another level, there were different ideas about how to find the point of intersection of the lines. Specifically, Akuti and Naomi used the table of values, whereas Martin used an algebraic approach.

Four ideas occurred in the conversation. The teacher gave another student, Pablo, an opportunity to restate what Naomi had said and also gave Naomi the chance to correct Pablo if that restatement did not reflect her ideas. The *orienting* move happened at the end, when the teacher summarized the different positions about the solution to the problem. This move was important because students could get lost when hearing different ideas. By orienting, the teacher is able to refocus the conversation and identify points of agreement and disagreement.

In the example provided, the teacher did not reach closure about the correct answer to the problem. One can expect that the teacher would achieve some closure afterward, when discussing the meaning of the values found for the x- and

Table 1 Several options are possible for performing discursive moves in a math class.

Discursive Move	Possible Questions or Comments to Pose
Request an explanation	"Maya, can you tell us more about the process of solving the problem?" "Jonathan, can you explain how you found the solution to that problem?" "Laura, how did you find your answer?" "Mark, can you tell us why you think that this answer makes sense?"
Orient	Alberto is arguing that he thinks the two lines do not intersect because there are no places in the table where the *x*- and *y*-coordinates are the same. Jonathan set up an equation and solved it algebraically and said that the two lines do intersect when $x = -2$. Is it possible that both people are correct? "Let's compare the method of using a table with the method of solving algebraically. Where could we locate our solution in the table?"

y-coordinates. The orienting move enables the teacher to present what has been said and contrast different positions. At the same time, orienting can help the class establish common ground. In the episode, the common ground involved expanding the table of values and looking for negative values of *x*. With orienting, a teacher can help students learn that they should go beyond identifying agreements and disagreements and work toward unpacking alternative perspectives.

USE DISCURSIVE MOVES IN MATH CLASS

The classroom episodes provide concrete examples of what a teacher can do to enact discursive moves to encourage students' reasoning and sense making through classroom discussions. Based on the discussion of the episodes, we created a list of possible questions that illustrate the two different discursive moves. **Table 1** provides a list of options that a teacher may use to promote reasoning and sense making.

RESEARCH TEACHER MOVES

Professional Standards for Teaching Mathematics (NCTM 1991) states expectations for mathematics teachers, one of which involves providing opportunities for students to engage in meaningful classroom discussions. Research supports the importance of mathematics teachers leading such discussions. Herbel-Eisenmann and Breyfogle (2005) suggest patterns of questioning that encourage students to discuss their strategies and solutions. Manouchehri and Enderson (1999) provide examples of productive mathematical discussions. Knuth and Peressini (2001) describe a framework to help make sense of the different roles of discourse.

This article attempts to provide examples to broaden the repertoire of possible actions that teachers can perform when leading class discussions (Chapin, O'Connor, and Anderson 2003; Herbel-Eisenmann and Cirillo 2009; Smith and Stein 2011). When students have the opportunities to discuss mathematical ideas in class, they develop more ownership of the mathematics that they learn and, moreover, they increase their understanding of mathematics (Ball and Bass 2003; Lampert and Cobb 2003). However, it is difficult for teachers to learn how to manage classroom discussions in meaningful ways (Chazan and Ball 1999). We expect that the examination of specific episodes can help teachers try some of these moves in their classroom.

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